Feasible implementation of a prediction algorithm for the game of roulette

Michael Small* and Chi Kong Tse
Department of Electronic and Information Engineering
Hong Kong Polytechnic University
Hong Kong
* Email: ensmall@polyu.edu.hk

***Abstract*— We present a mathematical model of the game of roulette and describe the implementation of an image processing and data analysis system to successfully predict the outcome of the game. Both the case of a fair (level) and biased (tilted) wheel will be described but the focus in this presentation is the (harder) case of a perfectly fair wheel. We show that when implemented on a casino-grade roulette wheel our technique obtains an expected return of over 40%, in contrast, the expected return for an uninformed gambler is -2.7%.**

I. Background

Unlike most casino games, the game of roulette is entirely mechanical and therefore deterministic. Of course the outcome of a spin depends with extreme sensitivity on initial conditions. Nonetheless, the spin of the wheel and the roll of the ball are perfectly well described by rather fundamental mechanical principles, and Newton's second law. In fact, the game was originally introduced in the mid-eighteenth century to provide gamblers with a sense of fairness. At the time, games of chance involved mainly cards and dice, and there was widespread discontent among gamblers that croupiers were capable of manipulating the outcome. Despite the perceived fairness and determinism in the game of Roulette, no successful technique to beat the house odds has ever been accurately described.

Nonetheless, reports of attempts to apply physical principles have been persistent and betray some remarkably similar features. The problem was considered by Henri Poincaré in his seminal work *Science and Method* [1]. Latter, techniques to predict the outcome were implemented and field tested by the Chaos collective [2] (described at length in [3]) as well as by Edward Thorp and Claude Shannon [4]. In 1969 Thorp published the only academic description of the possible procedure in a brief communication to the *Review of the International Statistical Institute* [5]. Similar experiments have also been (briefly) described by Epstein [6] in 1967. There have, of course, also been various attempts at capitalising on the predictability of Roulette by less exalted individuals, these are numerous and even less well documented.

A. Basics of the game

The game of roulette consists of a heavy wheel, machined and balanced to have very low friction and designed to spin for a relatively long time with a slowly decaying angular velocity. The wheel is spun in one direction, while a small ball is spun in the opposite direction on the rim of a fixed circularly inclined surface surrounding and abutting the wheel. As the ball loses momentum it drops toward the wheel and eventually will come to rest in one of 37 numbered pockets arranged around the outer edge of the spinning wheel. Various wagers can be made on which pocket, or group of pockets, the ball will eventually fall into. It is accepted practise that, on a successful wager on a single pocket, the casino will pay 35 to 1. Thus the expected return from a single wager on a fair wheel is $(35+1)\times\frac{1}{37}+(-1)\approx -2.7\%$ [7]. That is, for a given bet of $1, on average one will leave with $0.973, a loss of 2.7 cents/bet.

In the long-run, the house will, naturally, win. An American roulette wheel is even less fair and consists of 38 pockets (in addition to the standard 37 pockets, there is a second zero pocket, labelled "00"). We consider the European, 37 pocket, version as this is of more immediate interest to us [8]. Figure 1 illustrates the general structure, as well as the layout of pockets, on a standard European roulette wheel. The colour of the pockets (upon which one can wager alternate), but the ordering, although standard, adheres to no particular pattern.

Fig. 1. **The European roulette wheel.** The standard roulette wheel consists of 37 pockets (numbered from 0 to 36) on a heavy circular disk mounted on a central pivot (the central portion of the wheel). This rotates with low friction on a fixed metal frame (the *stator*) upon which are mounted several metal deflectors (on this model there are eight, one per eighth-sector of the wheel, mounted in the light wood panel rim). The sharp *frets* dividing the pockets on the rotating portion of the wheel introduce strong nonlinearity as the ball slows and bounces between pockets. The ball is shown in this figure at rest in pocket 32.

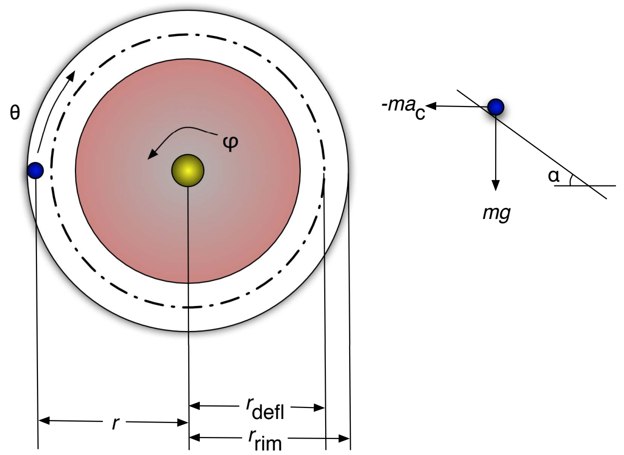

Fig. 2. **The dynamic model of ball and wheel.** On the left we show a top view of the roulette wheel (shaded region) and the stator (outer circles). The ball is moving on the stator with instantaneous position (r, θ) while the wheel is rotating with angular velocity $\dot{\varphi}$ (Not that the direction of the arrows here are for illustration only, the analysis in the text assume the same convention, clockwise positive, for both ball and wheel). The deflectors on the stator are modelled as a circle, concentric with the wheel, of radius r_{defl}. On the right we show a cross section and examination of the forces acting on the ball in the incline plane of the stator. The angle α is the incline of the stator, m is the mass of the ball, a_{c} is the radial acceleration of the ball, and g is gravity.

II. The algorithm

A. The model

As the ball spins in the rim of the roulette wheel it will gradually lose momentum. At some point in time, the centripetal force of the ball is precisely balanced by the downward (and inward) acceleration due to gravity. At this point the ball will begin to fall from the rim. A relatviely short period of time after that, the ball, having dropped from the rim, will hit one of the metal deflectors placed on the wheel. All motions up to this point are smooth and (in principle) easily predictable, we can therefore compute, from the initial angular position velocity and acceleration of the ball and the wheel (or even from their relative motion), the position of wheel when the ball hits a deflector, and therefore the section of the wheel closest to the collision. Of course, after this point, prediction becomes much more difficult, the motion of the ball is essentially chaotic, and we have insufficient information to predict it properly.

Fortunately, most casinos provide a rich array of options for betting on the outcome of a game of roulette. Using these various option we need only predict which quarter (or even which half) of the wheel the ball is likely to land in and bet on that. Hence, the problem is only to predict the trajectory of the ball from initial release until it's final resting place. Figure 2 highlights the relevant parameters.

Initially, the angular motion of the ball generates a centripetal accelaration which exceeds the force of gravtiy

$$\dot{\theta}^2 \quad > \quad \frac{g}{r}\tan\alpha. \tag{1}$$

When these two terms balance precisely, the ball will leave the rim. The time t_{rim} at which this occurs can be computed

$$t_{\mathrm{rim}} \quad = \quad -\frac{1}{\ddot{\theta}(0)}\left(\dot{\theta}(0) \pm \sqrt{\frac{g}{r}\tan\alpha}\right) \tag{2}$$

as can the corresponding position

$$\theta_{\mathrm{rim}} \quad = \quad \left|\frac{(\frac{g}{r}\tan\alpha) - \dot{\theta}(0)^2}{2\ddot{\theta}(0)}\right|_{2\pi} \tag{3}$$

where $|\cdot|_{2\pi}$ denotes modulo 2π. For some time the ball will continue to move freely around the wheel, during this phase the radial position of the ball decreases according to the difference between centripetal and gravitional forces according to the following second order differential equation

$$\ddot{r} \quad = \quad r\dot{\theta}^2\cos\alpha - g\sin\alpha. \tag{4}$$

Integrating (4) yields the position of the ball on the stator. Finally, we find the time $t = t_{\mathrm{defl}}$ for which $r(t)$, computed as the definite second integral of (4), is equal to r_{defl}. We can then compute the instantaneous angular position of the ball $\theta(t_{\mathrm{defl}})) = \theta(0) + \dot{\theta}(0)t_{\mathrm{defl}} + \frac{1}{2}\ddot{\theta}(0)t_{\mathrm{defl}}^2$ and the wheel $\varphi(t_{\mathrm{defl}})) = \varphi(0) + \dot{\varphi}(0)t_{\mathrm{defl}} + \frac{1}{2}\ddot{\varphi}(0)t_{\mathrm{defl}}^2$to give the salient value

$$\gamma \quad = \quad |\theta(t_{\mathrm{defl}}) - \varphi(t_{\mathrm{defl}})|_{2\pi} \tag{5}$$

denoting the angular location on the wheel directly below the point at which the ball strikes a deflector. Assuming that the deflectors are uniformly distributed around the rim,

The above model assumes that the table is level. It turns out that a very small deviation from level makes the game much more predictable. The reason for this (in our trials) is that by making the table not level the variation in force balance with position on the rim is such that the ball will almost always leave the rim in approximately the same region (a phenomenon which would be very easy to observe in practice). consequently, where and when the ball is likely to strike the deflector becomes much easier to predict. The only remaining challenge is to predict the position of the wheel at that time. Fortunately (for the errant gambler playing on a wonky wheel) the wheel is designed to rotate very smoothly and is therefore quite predictable.

B. Measurement

The problem of prediction is essentially two-fold. First, the various velocities must be estimated accurately. Given these estimates it is a trivial problem to then determine the point at which the ball will intersect with one of the deflectors on the stator. Second, one must then have an estimate of the scatter imposed on the ball by both the deflectors and possible collision with the individual frets (this problem we do not consider here, but it is potentially tractable). To apply this method *in situ*, one has the further complication of estimating the parameters r, r_{defl}, r_{rim}, α and possibly δ without attracting undue attention. We are not gamblers and therefore ignore this additional complication. Rather, we will assume that these quantities can be reliably estimated and restrict

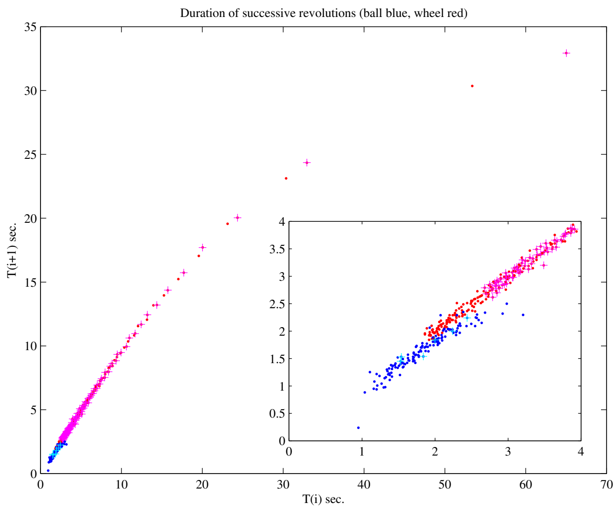

Fig. 3. **Hand-measurement of ball and wheel velocity for prediction.** From two spins of the wheel, and 20 successive spins of the ball we logged the time (in seconds) $T(i)$ for successive passes past a given point ($T(i)$ against $T(i+1)$). The red and pink dots depict these times for the wheel, the light and dark blue points are for the ball. A single trial of both ball and wheel is randomly highlighted with crosses (superimposed the pink and light blue points). The inset is an enlargement of the detail in the lower left corner. Both the noise and the determinism of this method are evident. In particular, the wheel velocity is relatively easy to calculate and decays slowly, in contrast the ball decays faster and is more difficult to measure. Using these (admittedly noisy) measurements we were able to successfully predict the half of the wheel in which the ball would stop in 13 of 22 trials (random with $p < 0.15$), yielding an expected return of $36/18 \times 13/22 - 1 = +18\%$ (a profit of \$0.18 for each \$1 bet). This trial run included predicting the precise location in which the ball landed on three occasions (random with $p < 0.02$).

Fig. 4. **Automated detection of ball and wheel position.** The figure depicts a single frame from the digital camera mounted above the roulette wheel. By comparing the motion of successive frames, we can deduce the location of the centre of the wheel (black cross) and the outer rim (yellow circle). The inner rim is a known fraction of the total diameter of the wheel and related to the physical dimensions (required to convert pixel measurements to SI-units). The boundary of the wheel is detected (light dotted white line) by looking for colour change in the image and should closely align with the outer yellow circle. The instantaneous position of the wheel is determined by processing the image to find the unique green "0" sector. The ball position is deduced by computing the difference between two frames in the rim region of the wheel.

our attention to the problem of prediction of the motion. To estimate the relevant positions, velocities and accelerations $(\theta, \dot{\theta}, \ddot{\theta}, \varphi, \dot{\varphi}, \ddot{\varphi})_{t=0}$ (or perhaps just $(\theta - \varphi, \dot{\theta} - \dot{\varphi}, \ddot{\theta} - \ddot{\varphi})_{t=0}$) we employ two distinct techniques.

Our first approach is to simply record the time at which ball and wheel pass a fixed point. This is a simple approach (probably that used in the early attempts to beat the wheels of Las Vegas) and is trivial to implement on a laptop computer, personal digital assistant, embedded system, or even a mobile phone[1]. Our results, depicted in figure 3, illustrate that the measurements, although noisy, are feasible. The noise introduced in this measurement is probably largely due to the lack of physical hand-eye co-ordination of the first author. Simple experiments with this configuration indicate that it is possible to accurately predict the correct half of the wheel in which the ball will come to rest.

Alternatively, we employ a digital camera mounted directly above the wheel to accurately and instantaneously measure the various physical parameters. In all our trials we use a regulation casino-grade roulette wheel (a 32" "President Revolution" roulette wheel manufacturer by Matsui Gaming Machine Co. Ltd., Tokyo). The wheel has 37 numbered slots (1 to 36 and 0) in the configuration shown in figure 1 and has a radius of 820 mm (spindle to rim). For the purposes of data collection we employ a Prosilica EC650C IEEE-1394 digital camera (1/3" CCD, 659×493 pixels at 90 frames per second). Data collection software was written and coded in C++ using the OpenCV library. It is data produced in this way which we analyse in the next section.

[1]Implementation on a "shoe-computer" should be relatively straightforward too.

C. *Data processing*

Figure 4 depicts a single frame captured by our camera (*in situ* of course it would be difficult to place a camera directly above the wheel, but this problem does not concern us here). Straightforward image processing tools allow us to detect both the position of the ball and wheel. The relevant techniques are described in Figure 4 caption. With this data we are able to estimate the relevant initial movements and then make our prediction. An example of such a calculation is given in Figure 5.

From 24 trials with this technique we predicted the collision with the deflector correctly (within $\pm 20°$ or arc) one quarter of the time, and within the correct quarter of the wheel ($\pm 45°$) half the time. After accounting for the final resting place of the ball we were able to predict the correct half of the wheel in which the ball fell on over two thirds of our trials. We are now in the process of repeating and extending these trials, and also refining the prediction algorithms further. Nonetheless, this technique confers a clear advantage: $\frac{36}{18} \times \frac{17}{24} - 1 = +41\%$. This technique confers an average return of \$1.41 for each \$1 bet, compared with the return on an uninformed wager of

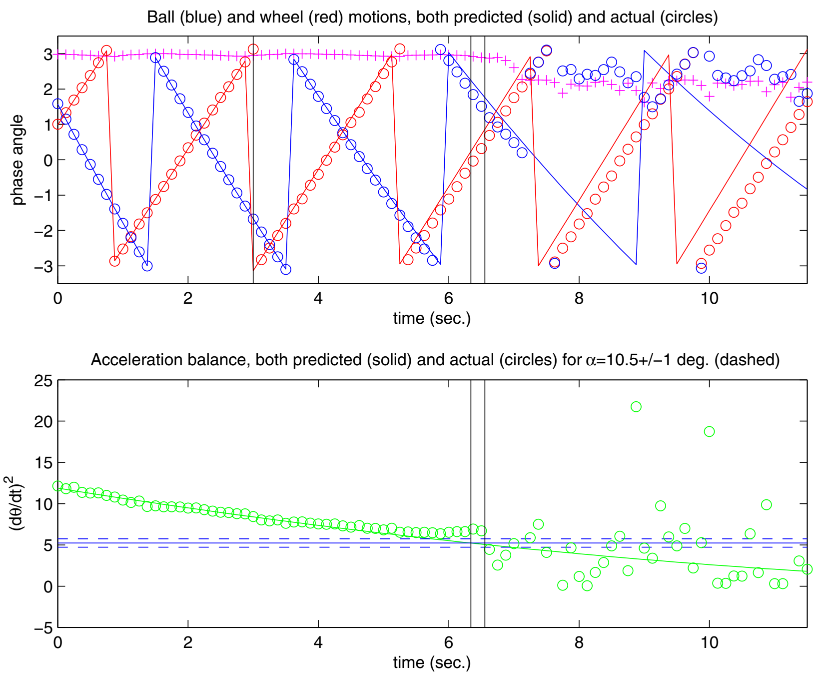

Fig. 5. **Computer based prediction.** The upper panel shows the measured position of the ball (blue circles, negative velocity) and wheel (red circles, positive velocity) together with the estimate produced from a least squares fit to the first three seconds of data (solid lines). Purple crosses correspond to the measure radial position of the ball, and have been rescaled to fit these axes. The lower panel shows the value of the left and right hand terms of equation (1). The right hand side of that equation is computed from physical properties of the wheel (but could be measured by observation), the left hand side of that equation is measured instantaneously (circles) and predicted from the first three seconds of the data (solid line). The intersection of these two lines, shown as two vertical black lines at around 6.2 sec., is the estimated time the ball will leave the outer rim, and then hit a deflector. In this simulation the result appears to be fairly accurate. Note that the estimation of the position of the ball fails once it comes to rest in the centre of the wheel, as the image processing methodologies we employ do not work well to detect the ball when it is moving with the background (wheel)

\$0.973 (including the initial stake).

III. Final Comments

There are several ways in which a determined gambler can improve their odds when playing roulette. We have focussed here on the investigation of that approach which we feel is scientifically most satisfying. A pragmatic gambler would be well advised just to look for a biased wheel and attempt to measure the relative angular position of ball and wheel. With the aid of a fairly basic computer, such a system could be made profitable. Playing a roulette wheel in situ with the aid of a digital camera would certainly be difficult, but various technical concerns exist: variable lighting as well as stationary of the observer and the angle of observation would all provide additional challenges. Nonetheless, a dexterous observer could, with practice, measure the relative position of ball and wheel during the initial spin and receive feedback from a small computing device on the predicted final position within time to legally place bets on that outcome. Of course, the variable location of the deflector will always make the outcome uncertain, and therefore one needs to be prepared for substantial drawdown.

Finally, this investigation has also highlighted several important step which the Roulette operators can take to ensure that their customers obtain minimum advantage. Unfortunately, we are unaware of to what extent these methods are already in force, and we have no space to discuss them further.

Acknowledgment

The authors would like to Prof. Marius Gerber for the originally introducing us to the dynamical systems connections in the game of roulette. Funding for this project was provided by the Hong Kong Polytechnic University under grant number 1BB-ZA. Experimental trials were conducted with the assistance of various undergraduate and final year project students.

References

[1] H. Poincaré, *Science and Method.* London: Nelson, 1914, english translation by Francis Maitland, preface by Bertrand Russell. Facsimile reprint in 1996 by Routledge/Thoemmes, London.
[2] J. D. Farmer and J. J. Sidorowich, "Predicting chaotic time series," *Physical Review Letters*, vol. 59, pp. 845–848, 1987.
[3] T. A. Bass, *The Newtonian Casino*. London: Penguin, 1990.
[4] E. O. Thorp, *The Mathematics of Gambling*. Gambling Times, 1985.
[5] ——, "Optimal gambling systems for favorable games," *Review of the International Statistical Institute*, vol. 37, pp. 273–293, 1969.
[6] R. A. Epstein, *The Theory of Gambling and Statisitcal Logic*. New York and London: Academic Press, 1967.
[7] F. Downton and R. L. Holder, "Banker's games and the gambling act 1968," *Journal of the Royal Statistical Society, series A*, vol. 135, pp. 336–364, 1972.
[8] B. Okuley and F. King-Poole, *Gamblers Guide to Macao*. Hong Kong: South China Morning Post Ltd, 1979.